Design of Antibacklash Pin-Gearing

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The paper suggests that pin gearing can be used for an antibacklash gear. The gear tooth curve derived from a cycloidal curve furnishes the pin gearing with kinematic antibacklash, low pressure angle, and no interference among other features. Pins on one side of a centerline are contacting the forward faces of a mating tooth gear. Simultaneously, pins on the other side of the centerline are contacting the backward faces of the gear. The correct center distance can be maintained in this multiple-contact mechanism by applying a force directed to each other center. A generating method of tooth-gear manufacture is also suggested. Contact and bending deformations as well as the gear stress are analyzed to help its design.

Key Words: Pin Gearing, Antibacklash Gear, Cycloidal Tooth Curve

1. Introduction

Pin gearing shown in Fig. 1 is one of the old devices for rotary power transmission (Drago, 1988). Cylindrical pins are on the pitch circle of a pin gear in a regular interval, and they are in mesh with a tooth gear. The pins are located between two circular disks with a center connected to a main shaft. Power can be transmitted to a pin gear from a tooth gear, or in the other way. However, as mentioned by Doughtie and James (1954), when a pin gear is given to the driver, injurious approaching action is brought forth. Therefore, a tooth gear is always given to the driver, and a pin gear to the driven for quiet action, since the power transmission during the recession period of the contact is smoother.

Pin gearing has a variety of characteristics different from involute gear, some of which are reviewed in this paper for new applications. Pin gearing appears to have been driven out of practice except few cases since the precision manufacturing and operations were not practical in old days. However, old devices may have a useful concept which can be utilized with advance of new technology.

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Pin gearing is suggested for antibacklash applications, for which its properties are to be fully exploited. Gear backlash is a lost uncontrolled gear motion during a directional change of rotation. Although the backlash does not affect unidirectional power transmission, it has a major effect on the precision of position-control machines such as robots. There can be many dimensional and operational tolerances affecting gear backlash. So it would be very difficult to control each cause of the backlash or to predetermine the backlash in involute gears. A variety of antibacklash devices are introduced in Kerkoff (1989) and Kamm (1990). One design of antibacklash gear is that two coaxial gears under an opposing torque with a screw or spring are meshing with a gear, in which one gear transmits a torque in one direction and the other gear in the other direction. Use of involute gear in this antibacklash design may cause an excessive wear on the gear surface. Furthermore, the screw or spring may have a hysteresis upon torque reversal. Pin gearing may also have some hysteresis due to the elastic compliance of gear tooth and pins. Thus, it is meaningful to study problems related to the elimination or reduction of gear backlash. Pin gearing has been tried for an antibacklash application in the patent of Kerkoff (1989), who suggested a circular-arc tooth profile. Moreover,

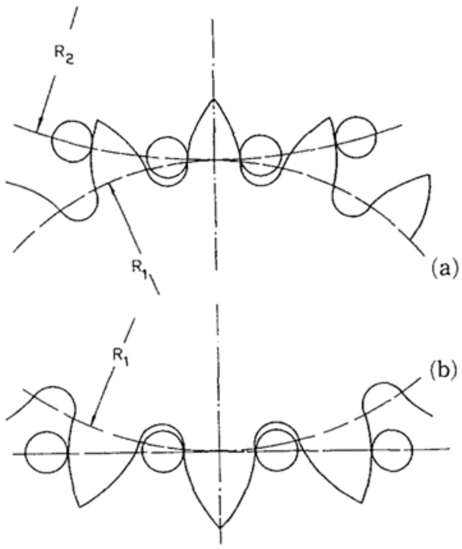

Fig. 1 A tooth gear in mesh with a pin gear (a) and a pin rack (b)

a toothed rack is replaced by a pin rack in Fig. 1 (b) for a gantry robot drive mechanism (Brown and Stewart, 1991), which can be an antibacklash device, even though it was not explicitly claimed there.

If involute gears are installed for antibacklash with both faces of a gear tooth to contact a mating gear, the mating gear teeth will bind to each other with a center distance error or any other dimensional changes. Binding of the teeth is a wedging action, which becomes evident with an increased coefficient of friction.

2. Kinematic Analysis

2.1 Tooth curve

In Fig. 2, the pitch-circle radius of the pin gear centered at O_2 is R_2, and its meshing tooth gear has a pitch-circle radius R_1 with a center at O_1. The dashed epicycloidal curve is generated with a rolling circle of radius R_2 on the pitch circle of radius R_1. The tooth curve of the solid line in Fig. 2 is in the distance of a pin radius R_3 from the dashed curve. Thus, the common normal at a contact point Q should be directed to the pitch point P. The tooth curve coordinates are given by Litvin (1994) in the Cartesian. They are rewritten here in the cylindrical coordinates (ρ, β) with

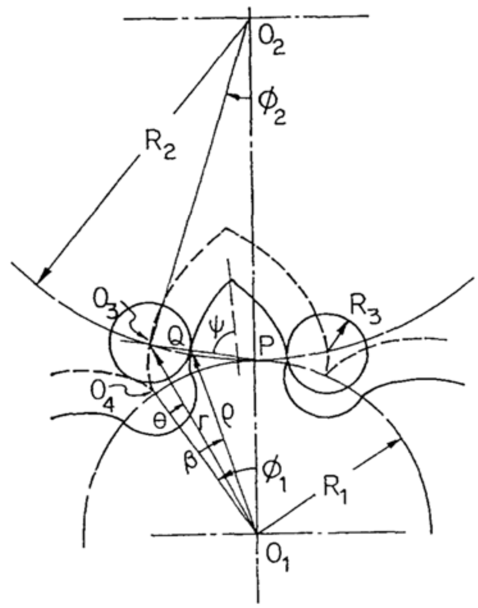

Fig. 2 A schematic diagram for the tooth curve in pin gearing

respect to the position angle ϕ_2 of the pin gear for later use.

$$\rho=\sqrt{\left[\left(2R_2\sin\frac{\phi_2}{2}-R_3\right)\cos\frac{\phi_2}{2}\right]^2+\left[\left(2R_2\sin\frac{\phi_2}{2}-R_3\right)\sin\frac{\phi_2}{2}+R_1\right]^2} \quad (1)$$

$$\beta=\frac{R_2}{R_1}\phi_2-\arctan\left[\frac{(2R_2\sin\frac{\phi_2}{2}-R_3)\cos\frac{\phi_2}{2}}{(2R_2\sin\frac{\phi_2}{2}-R_3)\sin\frac{\phi_2}{2}+R_1}\right] \quad (2)$$

The radius of curvature of the gear tooth profile is calculated in the following geometric relationship with above equations.

$$R=\frac{2R_1}{R_1+2R_2}\sqrt{\left(\frac{d\rho}{d\phi_2}\right)^2+\left(\rho\frac{d\beta}{d\phi_2}\right)^2} \quad (3)$$

The arc radius increases with an increasing angle ϕ_2. Cycloidal curve is determined with the radii of a pair of pitch circles of meshing gears. But the above equations include the pin radius, so in precise terms the tooth curve in pin gearing is not cycloidal in itself. As the pin-gear position angle ϕ_2 is increased from zero, the radial distance ρ decreases at the start, during which the contact is virtual. So the pin-gear angle at the beginning of pin-tooth contact upon passing the centerline is not zero but found with $d\rho/d\phi_2=0$ to be

$$\phi_{2i}=2\arcsin\frac{R_3(R_1+2R_2)}{4R_2(R_1+R_2)} \quad (4)$$

A unit tooth curve is obtained with Eqs. (1) and (2) as the angle ϕ_2 is incrementally increased until the angle β in Eq. (2) reaches a half tooth-interval angle, π/N_t, where N_t is the number of teeth. At the same time, the maximum angle $\phi_{2,max}$ is determined. This unit curve is repeated around the pitch circle of a tooth gear. Due to the symmetry about a centerline, a pin continues to contact gear teeth during the pin-gear rotation of an angle range $[-\phi_{2,max}, \phi_{2,max}]$ excluding a range $[-\phi_{2i}, \phi_{2i}]$. With a tooth-gear driver, a load-contact occurs during the second half. While a pin contacts a gear tooth by loading, another pin may also begin to contact another gear tooth by loading. Load-contacting pins vary in numbers with the gear position. The angle range of a loaded pin is divided by a pin-interval angle to give an average number of loaded pins, equivalent to the contact ratio (CR) in involute gears.

$$CR=\frac{\phi_{2,max}-\phi_{2i}}{2\pi/N_p} \quad (5)$$

where N_p is the number of pins in the pin gear. There is also an equal number of tooth-pin contacts on the other side of the centerline, through which power is not transmitted. With a pin radius $R_3==1.44R_1/N_t$, the contact ratios are calculated and shown in Fig. 3 for a few gear ratios with respect to the number of gear teeth. The contact ratio increases with an increasing number of pins.

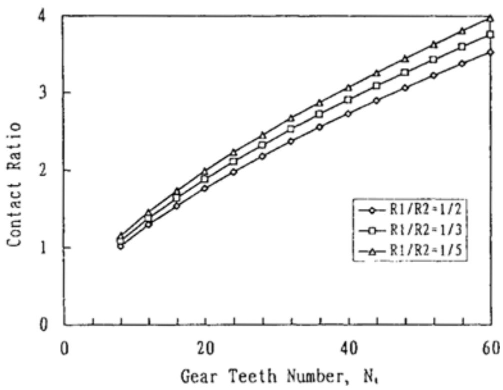

Fig. 3 Average number of power-transmitting teeth in pin gearing with a pin radius $R_3=1.44R_1/N_t$

A calculated example of a tooth curve is shown in Fig. 1(a) for $N_p=36$ and gear ratio $R_2/R_1=2$. In this multiple contact mechanism, each contacting pin has a pressure angle of half an angle between a centerline and the radial line to a pin center. Thus, the farther pin from a centerline will have the larger pressure angle. The pins on each side of the centerline are contacting the forward and backward faces of the gear teeth, respectively. Moreover, the entire contact path of the pin is symmetrical about the centerline, causing a positive pressure angle on either side of the centerline. Therefore, the pin gearing is antibacklash, for which the center distance must be kept constant. Due to the positive pressure angle on either side, the center distance of pin gearing with a contact ratio greater than one can not be less than the nominal value. Thus, the pin and tooth gears can be installed with a pushing force towards each center.

When the pitch radius of pin gear is increased to infinite to be a pin rack as shown in Fig. 1(b), its meshing tooth gear is an involute tooth gear, of which the base circle is also the pitch circle of the tooth gear in pin gearing, being tangent to the straight pitch line of the pin rack. Consequently, the pressure angle is zero, which may cause the difficulty in maintaining a correct distance between a pin rack and a tooth gear, since no pushing force can be sustained. In this case, an inverted design may be adopted with tooth rack and pin gear, which can be subjected to a force directed to each other. In the arrangement of Fig. 1(b), we may also suggest the manufacturing accuracy of involute tooth gears can be checked by the difference of the rotational displacement of the involute gear from the linear displacement of the pin rack, for which a new method is devised by Munro and Ling (1996).

2.2 Center-distance effect

Maintaining a correct center distance in pin gearing is a critical requirement for its proper operation. Thus, it would be helpful to discuss the effect of center-distance error in the gear performance. The pin gearing can be viewed as a cam-follower mechanism in which the driving tooth gear corresponds to a cam and the driven pin gear to a follower. Then the dashed line in Fig. 2 can be called a pitch curve of the cam, the cycloidal curve of a pin-center path, which can be obtained in Eqs. (1) and (2) with the pin-radius $R_3=0$. The cylindrical coordinates of the curve are denoted in (r, θ).

$$r=\sqrt{(R_2\sin\phi_2)^2+(R_1+R_2-R_2\cos\phi_2)^2} \tag{6}$$

$$\theta=\frac{R_2}{R_1}\phi_2-\arcsin\left(R_2 r\sin\phi_2\right) \tag{7}$$

The analysis about the pitch curve is carried out. Figure 4 shows a gear mesh with a center distance error e. While the tooth gear rotates a counterclockwise angle ϕ_3, the pin gear rotates a clockwise angle ϕ_4. These angles ϕ_3 and ϕ_4 correspond to the angles ϕ_1, ϕ_2 in Fig. 2, respectively, without an error, $e=0$. The pin gear contacts the tooth gear of $\phi_3=0$ at the pitch curve point of coordinates (r_o, θ_o), with a pin-gear angle ϵ, in the following relationships of geometry in Fig. 4.

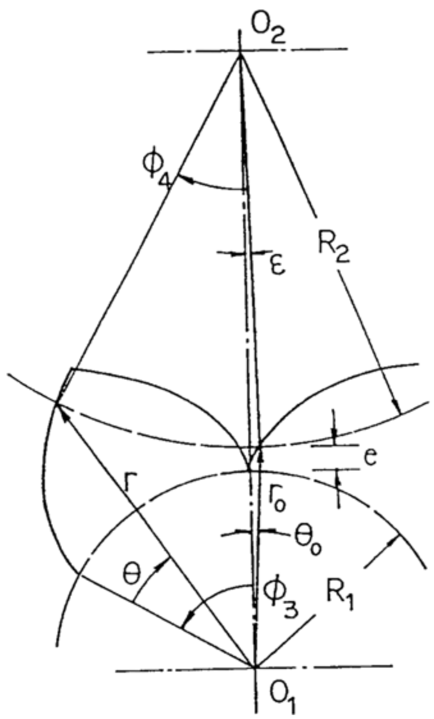

Fig. 4 A schematic diagram for the kinematic analysis of pin gearing with a center-distance error

$$R_2\sin\epsilon=r_o\sin\theta_o \tag{8}$$

$$R_2\cos\epsilon+r_o\cos\theta_o=R_1+R_2+e \tag{9}$$

These are solved with the Eqs. (6) and (7) in trial and error for the values r_o, θ_o, and e. When the pin gear contacts a tooth gear at the coordinates (r, θ) of Eqs. (6) and (7) beyond (r_o, θ_o), the tooth and pin gears come to the following angular position.

$$\phi_3=\theta+\arccos\left[\frac{(R_1+R_2+e)^2-R_2^2+r^2}{2r(R_1+R_2+e)}\right] \tag{10}$$

$$\phi_4=e+\arcsin\left[\frac{r}{R_2}\sin(\phi_3-\theta)\right] \tag{11}$$

A lag angle of the pin gear due to a center-distance error is $\epsilon-\phi_4+R_1\phi_3/R_2$, which varies smoothly with a gear rotation. The lag angle is defined so further rotation of a pin gear by the lag angle will bring the pin gear to the correct position without a center distance error. When two neighboring pins have different lag angles, only the pin of a lower lag angle will contact the gear tooth. The transfer of loading from a pin to the next pin occurs with an equal lag angle, which is about $\phi_3=-2°$ or $\phi_3=28°$ in calculated examples of Fig. 5. A pin contacts the gear tooth during the pin-gear rotation from $\phi_3=-2°$ to $\phi_3=28°$. Thus, in the pin gearing with a center-distance error, only a single contact occurs between the pin gear and tooth gear. Disadvantages of no multiple contacts in pin gearing are obvious. Therefore, the center distance must be kept correct not only for the antibacklash but also for the load sharing

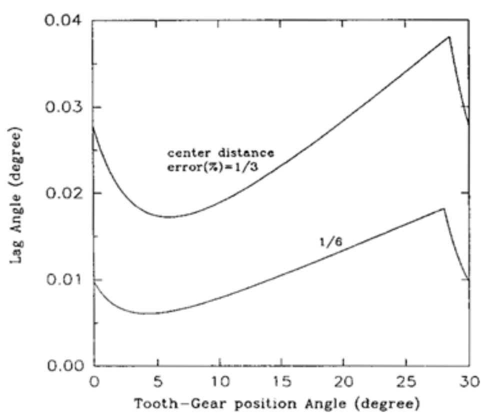

Fig. 5 Lag angle of a pin gear with a center-distance error in the 24-pin and 12-tooth gears

of multiple contacts. The backlash can be calculated directly from the lag angle.

3. Gear Stress

Both the tooth-bending stress and gear-surface contact stress must be considered for the strength design of gear tooth. Approximate solutions are provided for the stresses at each critical loading point.

3.1 Gear-tooth bending stress

Maximum bending stress occurs to a gear tooth just before another pin begins to deliver a torque, at which time the pin-gear is at a position angle ϕ_{2t}.

$$\phi_{2t}=\frac{2\pi}{N_p}+\phi_{2i} \tag{12}$$

This position corresponds to the minimum number of loaded contacts. Unlike the involute gear, the tooth gears in pin gearing vary in shape, depending on gear ratio and pin size in addition to tooth interval distance. Especially for the gears of a gear ratio $R_2/R_1<1$, tooth-shape change is significant. However, the shape change is minor for the gears of a gear ratio $R_2/R_1>2$, which is a more practical case in reduction gears. For example, with $N_t=12$ and $N_p=24$, the tooth length has about 2.5% difference from the involute-tooth length of $R_2/R_1\to\infty$. Thus, it is considered that tooth stresses can be calculated with one tooth form for each tooth-interval and pin size by neglecting minor shape difference caused by a different gear ratio. For the tooth gear under a normal contact force F per unit gear face width, the maximum bending stress can be expressed in a Lewis-form equation, similar to the involute spur gear.

$$\sigma_b=\frac{F\cos(\phi_{2t}/2)}{(2R_1/N_t)\,Y} \tag{13}$$

Here a Lewis-form factor Y includes a stress concentration effect, which was calculated in a finite element analysis of a mesh with total nodes more than 10000 as shown in Fig. 6. The mesh was constructed for about two tooth-intervals, with the load-free side teeth being excluded for simplicity, and the lower boundaries are made fixed. The loaded tooth surface has a fine mesh of one third element size of the near inner-mesh element. Maximum tensile stress occurs on the circular root surface below the pitch circle with a minimum tooth thickness. Even though a compressive stress on the root surface is larger than the tensile stress, the former has a marginal effect on tooth integrity. The calculated examples for Y with $N_t=12$ and $R_3=1.44R_1/N_t$ are given in Table 1 for a few cases of gear ratio.

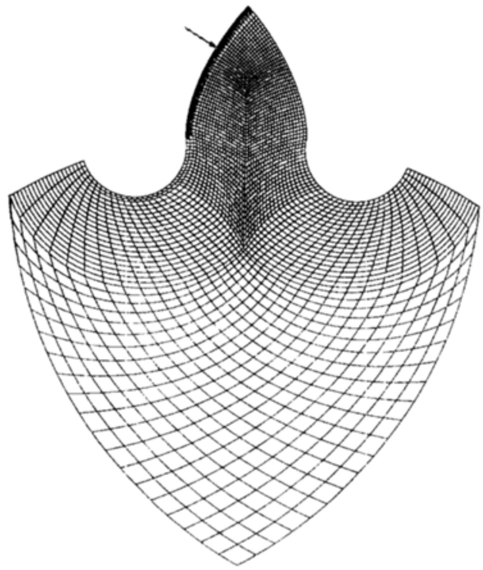

Fig. 6 A finite element model for the calculation of tooth-gear stress

Table 1 Lewis-form factor for bending stress in pin gearing

N_t	N_p	Y
12	24	0.332
12	36	0.354
12	48	0.365
12	60	0.371

3.2 Gear surface contact stress

The pin-tooth contact stress can be obtained in Young (1989) by assuming a parallel-cylinder contact, with a pin of radius R_3 on a surface of arc radius R, in the material of Young's modulus

E and Poisson's ratio ν.

$$\sigma_c = 0.798\sqrt{\frac{F(R_3+R)E}{4R_3R(1-\nu^2)}} \tag{14}$$

The arc radius R of Eq. (3) increases as the angle ϕ_2 increases, so the contact stress would be maximum at the initial position ϕ_{2i}, when a gear tooth begins to deliver a torque to a pin. This can be a major check for the design stress requirement in the comparable sizes of pin diameter and tooth thickness. When the contact stress causes pitting-fatigue failure, the contact stress can be reduced by delaying the torque transmission, that is, by making the angle ϕ_{2i} greater than the value in Eq. (4). The force F in Eq. (14) is not a total transmitting force but a sharing force with a neighboring pin since the contact ratio in Eq. (5) is greater than 1.When a rigid pin is assumed, the contact stress is multiplied by $\sqrt{2}$.

4. Deformation Analysis

Even though pin gearing with a correct center distance is kinematically antibacklash, the elastic deformation of gear tooth may induce a backlash, which must be considered in the design of a precision gear. For example, pins can be made to have a radius increased by an expected backlash. Thus, discussions are followed about the Hertz contact and tooth bending deformations, which are added for a total deflection. In order to minimize both the bending deformation of the pins and pin-hole wear, high-rigidity pins can be used with a dimensional tolerance absorbed by a gear tooth, for which rigid pins are assumed in the analysis.

4.1 Contact deformation

For an elastic arc surface pressed by a rigid cylindrical roller, no direct solution of elastic indentation seems to be available. We start from a related solution (Timoshenko and Goodier, 1970) about a two-dimensional problem of uniform pressure q on a segment of width $2a$ shown in Fig. 7, where d is a distance to a free-surface boundary. The deflection at the load center is

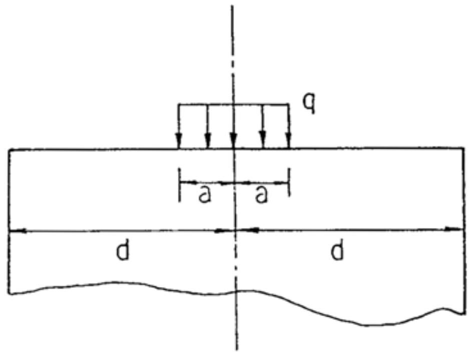

Fig. 7 A cross section of a uniform pressure on a partial plane

$$v_c = \frac{2aq}{\pi E}(b - 2\ln a) \tag{15}$$

where $b = 2\ln d + (1-\nu)$. When two cylinders of parallel axes are pressed to each other by a force F, the pressure distribution is given in Landau and Lifshitz (1970).

$$p = \frac{2F}{\pi a}\sqrt{1-(x/a)^2} \tag{16}$$

where $2a$ is the width of contact with a center at $x=0$ and given as $a = \sqrt{8(1-\nu^2)FR_3/(\pi E)}$ for a rigid roller of radius R_3 on a flat plane. Taking an incremental pressure dp in Eq. (16), the center deflection is calculated by integrating each incremental deflection.

$$\begin{aligned} v_c &= \int \frac{2x(b-2\ln x)}{\pi E}dp \\ &= \frac{F}{\pi E}(b - 2\ln a + 0.3863) \end{aligned} \tag{17}$$

The above two cases show that the center deflection from a segmental uniform pressure is almost identical to that of a pressing rigid roller when a contact width is far smaller than the plane size, $2\ln(d/a) \gg 0.3863$. This observation is extended to a pin contacting a gear tooth surface of an arc radius. Thus, a uniform pressure $q = F/2a$ is applied on a segment of cylindrical surface of radius R in parallel to the axis. The configuration is similar to Fig. 7, but the lower plane has an arc radius R. Finite element analyses are conducted for various radii, and an approximate solution is obtained in curve-fitting to the form of Eq. (15).

$$v_c = \frac{2aq}{\pi E}\left(1.81\ln\frac{R}{a} + 1.32\right) \tag{18}$$

Here we use the contact width a of a rigid roller of radius R_3 pressing on an elastic cylinder of radius R, given in Landau and Lifshitz (1970), since the contact width is considered far smaller than the roller radius.

$$a=\sqrt{\frac{4(1-\nu^2)}{\pi E}F\left(\frac{RR_3}{R+R_3}\right)} \tag{19}$$

4.2 Tooth bending

At the gear-surface contact point in Fig. 2, the common normal line of action makes an angle ψ with a radial tooth-symmetry line.

$$\psi=\left(\frac{1}{2}+\frac{1}{N_t}\right)\pi-\left(\frac{1}{2}+\frac{R_2}{R_1}\right)\phi_2 \tag{20}$$

With this angle, the tangential component of the normal force F for gear-tooth bending is $F_t=F\sin\psi$, and the radial compressive force is $F_r=F\cos\psi$, and the latter is considered to have a marginal effect on the tooth deformation. Maximum deflection occurs at a pin-gear position angle just before ϕ_{2t}, where the load-point distance from a tooth bottom is $s=\rho\cos(\pi/N_t-\beta)-(R_1-R_3)\cos(\pi/N_t)$. Denoting a distance from the tooth bottom in x and the tooth thickness $2h$, the tooth deflection in the direction of force F is calculated in the Castigliano method.

$$v_b=\frac{3F\sin^2\psi}{2E}\int\frac{(s-x)^2}{h^3}dx \tag{21}$$

where ψ can be approximated in Eq. (20) with $\phi_2=\phi_{2t}$. The integration is performed numerically with following relationships.

$$h=\rho\sin\left(\frac{\pi}{N_t}-\beta\right) \tag{22}$$

$$x=\rho\cos\left(\frac{\pi}{N_t}-\beta\right)-(R_1-R_3)\cos\left(\frac{\pi}{N_t}\right) \tag{23}$$

For the addendum portion of gear, ρ and β of Eqs. (1) and (2) are used with an angle range from ϕ_{2i} to ϕ_{2t}, and for the tooth-root part the followings are used with a range from $\gamma=0$ to γ that gives the angle β at ϕ_{2i} of Eq. (4).

$$\rho=\sqrt{R_3^2-2R_1R_3\cos\gamma+R_1^2} \tag{24}$$

$$\beta=\arctan\left(\frac{R_3\sin\gamma}{R_1-R_3\cos\gamma}\right) \tag{25}$$

With a pin radius $R_3=1.44R_1/N_t$, the calculated

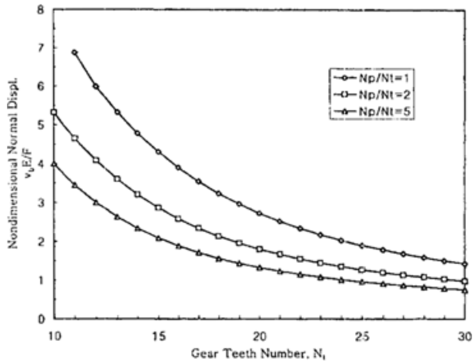

Fig. 8 Maximum normal displacement at the contact point due to bending

examples are shown in a nondimensional form of v_bE/F in Fig. 8 for each gear ratio and N_t.

5. Tooth Gear Manufacture

An advantage of involute gear is that the cutting tool has a simple shape of flat cutting faces of rack-gear in the generating method of the gear manufacture. The same advantage can be maintained for pin-gearing. As the tooth curve in pin gearing varies according to the pin radius and gear size and ratio, the manufacturing cost could be high without specialty tools. Here we suggest a generating method that tooth gears can be machined with a device similar to a pin-gear shape. At first, multiple cylindrical cutters of a diameter equal to the pin diameter are located at the pin positions around a pin gear. The cutters are made to rotate about each center as they are supported by a pin-gear disk. Then a gear blank for a tooth gear is brought to the position of a tooth gear in mesh with the pin gear. Both the gear blank and the pin-gear disk are rotated synchronously about each center in a predetermined angular velocity ratio.

In addition, an involute curve is also a cycloidal curve with a generating circle of infinite radius. Thus, rotating cylindrical cutters are positioned along a straight pitch line for an involute gear, shown in the pin rack of Fig. 1(b). A gear blank is brought to the position of the tooth gear so that the pitch line of the pin rack is tangent to the base circle of the involute gear.

Then the gear blank is rotated about its center while the pin rack is moved along the pitch line so that the tangent point may have a same velocity.

6. Conclusions

The paper draws close attention to pin gearing including kinematics, stress and deformation analysis, and generating method of tooth gear manufacture, with an objective of its antibacklash gear design. Kinematically, the pitch circle of the pin gear is also a generating circle for the cycloidal curve of a meshing tooth gear. The tooth curve is in the distance of a pin radius from the cycloidal curve.

Pin gearing has many characteristic features. One of them is low pressure angle. Forces are transmitted to contacting pins on one side of a centerline, with a pressure angle of half the angle between the centerline and the radial line to the pin center. The first pin from the centerline is subjected to a major force, and the pressure angle is less than half an interval angle between pins, for instance 5° in 36-pin gear. Low pressure angle brings about a low gear-separating force. However, as the pressure angle varies with gear rotation, the tangential force for transmitting power also varies, which can be detrimental. The force variation can be minimized with an increased number of pins, for example, less than 1 % for a pin gear with more than 24 pins.

Pin gearing has another advantage of no interference between gear teeth and pins. So the number of the teeth can be reduced without causing interference, which also increases tooth thickness. Thus, in a pin-gearing design, there has to be some trade-off between the increase of tooth thickness and the increase of number of pins.

It is shown that the performance of pin gearing deteriorates significantly with a center distance error. Thus, the center distance must be kept correct for the pin gearing to be properly operative. The correct center distance can be maintained by using the property that the pin gear has no interference and that the center distance of the gear mesh with a positive pressure angle can not be less than a correct distance. When meshing gears are applied with a force directed to each center, the contacting pins on both sides of the centerline are subjected to forces which prevent the gear from advancing. Pin gearing is a multiple-contact mechanism, only if a correct center distance is kept. Therefore, pin gearing can be installed so they are subjected to a force directed to each other center, large enough to balance a gear-separating force developed during power transmission.

Approaching pins on one side of a centerline are contacting the backward faces of gear teeth. Simultaneously, the recessing pins on the other side of the centerline are contacting the forward faces of the gear teeth. Forces are transmitted to the pins on the forward faces. Upon reversal of the tooth-gear rotational direction, the forward and backward gear faces switch without a loss of contact. As a result, pin gearing can be antibacklash. However, the backlash induced by the contact and bending deformations must be minimized during the operations.

Acknowledgements

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